21. $f(x) = 9x^{2/3} + 3x - 6$

$f'(x) = \frac{6}{x^{1/3}} + 3 = 0$

$x = -8$

$f'(x) = *$ $x = 0$

$- \quad * \quad +$

8

$f''(x) = -\frac{2}{x^{4/3}}\Big|_{-8} < 0$ $\frown$ max at $x = -8$

22. $\int_0^4 x\, f'(x)\, dx = x f(x)\Big|_0^4 - \int_0^4 f(x)\, dx$

$u = x \quad f'(x)dx = dv$

$du = dx \quad f(x) = v$

$= 4f(4) - 0 - 8$

$= 4(-3) - 8 = -20$

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23. $\text{slope} = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$x = r\cos\theta = 2\theta\cos\theta$

$y = r\sin\theta = 2\theta\sin\theta$

$r = 2\theta$

$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\theta\cos\theta + \sin\theta \cdot 2}{-2\theta\sin\theta + \cos\theta \cdot 2}\Big|_{\theta = \frac{\pi}{2}} = \frac{2}{-\pi}$

$0 \quad + \quad 2$

$-\pi \quad + \quad 0$

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26. $\sum_{n=0}^{\infty} a_n (x-2)^n$

$\frac{a_{n+1}}{a_n} = \left(\frac{2n+1}{3n-1}\right) \cdot \frac{a_n}{a_n}$

$a_n = \left(\frac{2n+1}{3n-1}\right) a_{n-1}$

ratio test $\lim_{n\to\infty} \left| \frac{a_{n+1}(x-2)^{n+1}}{a_n (x-2)^n} \right|$

$\lim_{n\to\infty} \left| \left(\frac{2n+1}{3n-1}\right)(x-2) \right| = \frac{2}{3}|x-2| < 1$

$|x-2| < \frac{3}{2}$

center, radius

$-\frac{3}{2} < x-2 < \frac{3}{2}$

$2-\frac{3}{2} < x < 2+\frac{3}{2}$

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28. $f(x) = \sin\left(\frac{x+1}{x^2}\right)$

Yes I $\lim_{x\to\infty} \sin\left(\frac{x+1}{x^2}\right) = 0$ HA $y=0$

NO II

no III vert. asymptote

$-1 \le \sin u \le 1$ $\lim_{x\to a} f(x) = \pm\infty$

$x=0$ $\lim_{x\to 0} \sin\left(\frac{x+1}{x^2}\right) \le 1$

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Review 21 Improper integrals

$\int_a^\infty f(x)\,dx$, $\int_{-\infty}^a f(x)\,dx$, $\int_{-\infty}^\infty f(x)\,dx$

$\int_a^b f(x)\,dx$ improper if $f(x)$ is $\not\in$ on $[a,b]$

example $\int_{-2}^{2} \frac{1}{x^2}\,dx$

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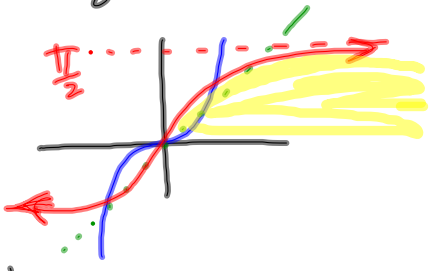
make it proper:

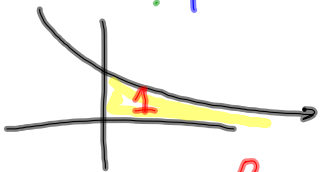
conv or (div)

$\int_0^\infty \tan^{-1}x\,dx = \lim_{b\to\infty} \int_0^b \tan^{-1}x\,dx$



$\int_0^\infty \frac{1}{e^x}\,dx$

$\lim_{b\to\infty} \int_0^b e^{-x}\,dx$



$\lim_{b\to\infty} \left(-e^{-x}\Big|_0^b\right) = \lim_{b\to\infty} \left(-\frac{1}{e^b} + \frac{1}{e^0}\right) = 1$

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make it proper $\int_{-2}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$

$\int_{-2}^{-1} + \int_{-1}^{0} + \int_{0}^{1} + \int_{1}^{2}$

does it converge?

direct comparison

$\int_1^\infty \frac{1}{1+x^3}\,dx < \int_1^\infty \frac{1}{x^3}$

limit comparison

if both functions grow at the same rate the integrals behave the same

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